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Viscosity approximation methods for a common fixed point of finite family of nonexpansive mappings

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Abstract

Let K be a nonempty closed and convex subset of a real Banach space E. Let $T : K \to E$ be a nonexpansive weakly inward mapping with $F(T) \neq \emptyset$ and $f : K \to K$ be a contraction. Then for $t \in (0,1)$, there exists a sequence $\{y_t\} \subset K$ satisfying $y_t = (1-t)f(y_t) + tT(y_t)$. Furthermore, if E is a strictly convex real reflexive Banach space having a uniformly Gâteaux differentiable norm, then $\{y_t\}$ converges strongly to a fixed point p of T such that p is the unique solution in $F(T)$ to a certain variational inequality. Moreover, if $\{T_i,\ i = 1,2,\ldots,r\}$ is a family of nonexpansive mappings, then an explicit iteration process which converges strongly to a common fixed point of $\{T_i,\ i = 1,2,\ldots,r\}$ and to a solution of a certain variational inequality is constructed. Under the above setting, the family $T_i,\ i = 1,2,\ldots,r$ need not satisfy the requirment that $\bigcap_{i=1}^{r} F(T_i) = F(T_rT_{r-1},\ldots,T_1) = F(T_1T_r,\ldots,T_2) =,\cdots,= F(T_{r-1}T_{r-2},\ldots,T_1T_r)$.

Keywords: Nonexpansive mappings; Strictly convex spaces; Weakly inward maps

1. Introduction

Let K be a nonempty subset of a normed linear space, E. A mapping $T : K \to E$ is called *nonexpansive* if $\|Tx - Ty\| \leqslant \|x - y\|$ for all $x, y \in K$. In [5], Kirk introduced an iterative process given by

$$x_{n+1} := a_0x_n + a_1Tx_n + a_2T^2x_n +, \cdots, +a_rT^rx_n, \tag{1.1}$$

where $a_i \geqslant 0$, $a_0 > 0$ and $\sum_{i=0}^{r} a_i = 1$, for approximating fixed points of nonexpansive mappings on convex subset of uniformly convex Banach spaces. Maiti and Saha [9] extended the results of Kirk [5].

Let K be a nonempty closed convex and *bounded* subset of a real Banach space E. Let $T_i : K \to K (i = 1,2,\ldots,r)$ be nonexpansive mappings and let

$$S := a_0I + a_1T_1 + a_2T_2 +, \cdots, +a_rT_r, \tag{1.2}$$

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