INSTITUTE OF PHYSICS PUBLISHING SMART MATERIALS AND STRUCTURES

Smart Mater. Struct. **12** (2003) N1–N4 PII: S0964-1726(03)61738-6

TECHNICAL NOTE

Review of the mechanics of materials models for one-dimensional surface-bonded piezoelectric actuators

Bandar A Alzahrani and Abdulmalik A A Alghamdi

Mechanical Engineering Department, King Abdulaziz University, PO Box 80204, Jeddah 21589, Saudi Arabia

Received 25 January 2002, in final form 26 March 2003
Published 28 April 2003
Online at stacks.iop.org/SMS/12/N1

Abstract

This note reviews the commonest and simplest theoretical models used in modelling one-dimensional smart structures. These models can be used for any type of induced strain; however, the piezoelectric actuator is used here as a typical active element. A numerical example is given to show the differences among these models especially as regards the strain induced in the beam.

1. Introduction

The concept of smart or adaptive structures was introduced in the 1980s by researchers in composite materials, structural dynamics and vibration. The need for active structures goes back to the need for structure that can change itself in an appropriate manner. By introducing intelligent materials, such as piezoceramics and shape memory alloys, it was possible to actively change the behaviour of a passive structure with the help of these intelligent materials. With the advances in composite materials and technology, adaptive structures became reality, where the actuators are embedded within the structure or surface bonded to it. Then the concept of smart structure emerged for structures with sensors, actuators, power supplies and control circuits integrated in the structure and participating in the function of the structure as well. The aim of this note is to review the kinematics models proposed in the literature.

2. The uniform strain model (USM)

This model was introduced by Bailey and Hubbard (1985). The actuation strain is assumed to be uniform in the piezoelectric actuator. In addition, the induced strain in the beam is assumed to be uniform as well. This assumption is acceptable for actuators embedded at the middle of the beam, but not for surface-bonded ones. However, it was proposed in the literature in this way, with ignorance of the flexural stiffness of the beam. This model is referred to as the simple model or uniform strain model (USM) in the literature. Looking at this model, the uniform strain in the beam and the actuator, shown in figure 1, is given by

$$\varepsilon = \varepsilon_a = \frac{\Lambda}{1+\Psi} \tag{1}$$

where Ψ is the stiffness ratio, given as

$$\Psi = \frac{Et}{E_a t_a}, \tag{2}$$

and Λ is the free strain caused by the piezoelectricity effect, given by

$$\Lambda = \frac{V d_{31}}{t_a} \tag{3}$$

where E is the modulus of elasticity of the beam, E_a is the modulus of elasticity of the actuator, t_a is the actuator thickness, t is the beam thickness, V is the electrical potential applied across the actuator electrodes and d_{31} is the piezoelectric electromechanical coupling.

3. The pin force model (ULM)

In the pin force model, the strain is assumed to be uniform in the actuator and linear in the beam. Sometimes this model is referred to as the uniform–linear strain model (Alghamdi and Dasgupta 2000). The difference between the USM and the pin

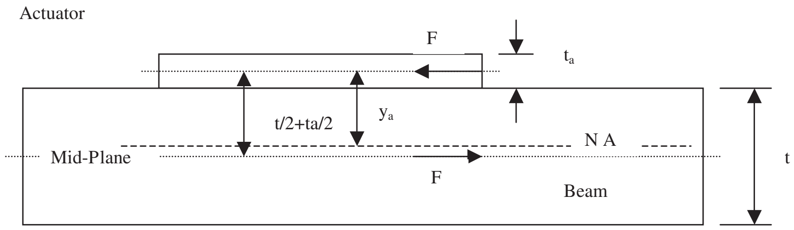

Figure 1. An actuator bonded to a one-dimensional beam.

force model lies in the flexural stiffness term that is added here for the passive beam.

One can write the strain in the actuator as (Crawley and de Luis 1987),

$$\varepsilon_a = \frac{4\Lambda}{4+\Psi}. \tag{4}$$

For constant stiffness ratio, the pin force model predicts more actuator strain as compared to the USM. Similarly, the strain developed in the beam is linear, given by

$$\varepsilon = \frac{\Lambda}{4+\Psi}\left(\frac{t+6y}{t}\right) \tag{5}$$

where y is measured from the beam mid-plane.

4. The enhanced pin force model (LSM)

This model was developed by Chaudhry and Rogers (1994); they assumed linear distribution of the strain in both the actuator and the beam. Thus, the flexural stiffness of the actuator is taken into account. One can write the strain distribution in the actuator as

$$\varepsilon_a = \Lambda\left(1 - \frac{\Psi}{4(1+\Psi)}\frac{t_a+6y_a}{t_a}\right) \tag{6}$$

where y_a is measured from the actuator mid-plane. Similarly, the strain distribution in the beam is given by

$$\varepsilon = \frac{\Lambda}{4(1+\Psi)}\left(\frac{t+6y}{t}\right) \tag{7}$$

where y is measured from the beam mid-plane.

5. The Bernoulli–Euler model (BEM)

In this model, composite material analysis is used to predict the response of the beam due to the surface-bonded actuator. Classical laminate plate theory for beams is used here, assuming no external loading; only electrical loads are considered. The system of linear matrices is solved symbolically for the uniform strain (ε_0) and curvature (κ). The beam and the actuator are treated as a one-dimensional structure with a perfect bond. The total strain in the structure can be written as (Crawley and de Luis)

$$\varepsilon = \varepsilon_0 - z\kappa \tag{8}$$

where z is the distance measured from the structure mid-plane. Using classical laminate plate theory, the uniform strain is given as

$$\varepsilon_0 = \frac{(T^2+3+\Psi+3T)\Lambda}{\alpha} \tag{9}$$

where T is the thickness ratio ($T = t/t_a$) and α is

$$\alpha = (4+\Psi)T^2 + 4 + 6T + \frac{1}{\Psi}. \tag{10}$$

The curvature (κ) is written as

$$\kappa = \frac{6(1+T)T\Lambda}{t\alpha}. \tag{11}$$

6. The strain energy model (SEM)

This model was developed by Wang and Rogers (1991). It is based on three assumptions: uniform strain in the actuator, linear strain in the beam and zero strain at the lower surface of the beam, for the configuration considered. The last assumption may not be appropriate for a surface-bonded configuration. The zero-strain case is a specific case that can be obtained for a certain actuation strain and specific stiffness ratio. One can obtain the uniform strain in the actuator as

$$\varepsilon_a = \frac{6\Lambda}{6+\Psi}. \tag{12}$$

In addition, the strain distribution in the beam is given as

$$\varepsilon = \frac{6\Lambda}{6+\Psi}\frac{1}{t}\left(\frac{t}{2}+z\right) \tag{13}$$

where z is measured from the beam mid-plane.

7. Results and discussion

Consider an aluminium beam with modulus of elasticity $E = 70$ GPa, width $w = 50$ mm and thickness $t = 10$ mm, to be used as the passive beam. Assume the piezoelectric ceramic actuator type PZT-5H, with thickness $t_a = 1$ mm, width (the same as that of the beam) $w_a = 50$ mm, elasticity modulus $E_a = 64$ GPa and electromechanical coupling term $d_{31} = -274 \times 10^{-12}$ m V^{-1}.

Figure 2 shows the normalized induced strain at the contact surface between the actuator and the beam versus thickness ratio (T). The induced strain is normalized with respect to the free strain Λ. For zero thickness ratio the actuator is considered to be free and surface bonded to the air; thus the induced strain is maximum and equal to the free strain (Λ) value. As expected, induced strain decreases with increase in the thickness ratio. In other words, the actuator becomes less effective with increase of the beam thickness or with decrease in the actuator thickness. The prediction of the enhanced pin force model is identical to that of the USM at the interface layer; thus, only the USM is plotted. The pin force model

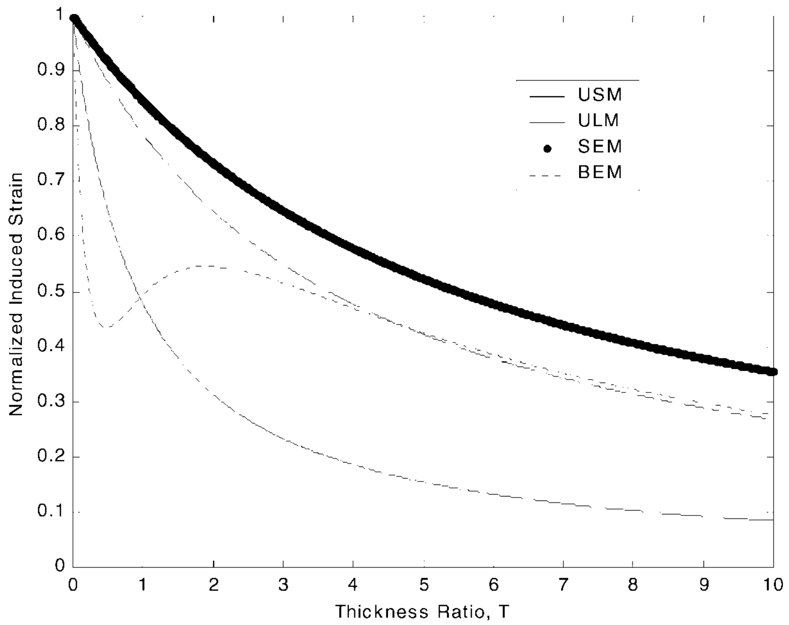

Figure 2. The relation between the normalized induced strain and the thickness ratio.

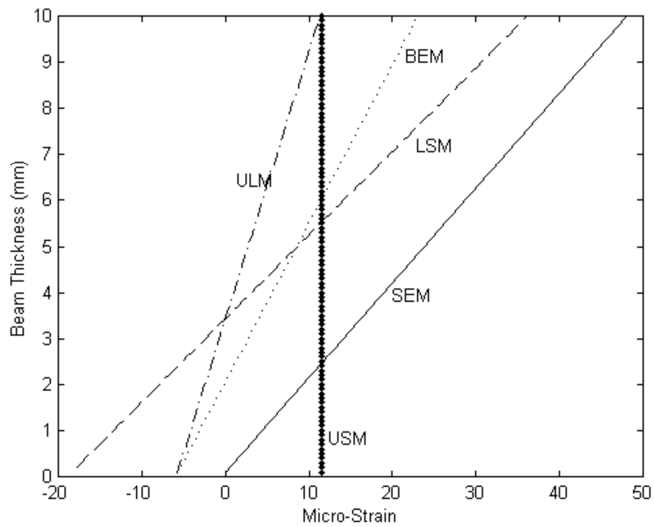

Figure 3. The induced strain distribution in the beam as predicted by the proposed models.

agrees very well with the Bernoulli–Euler model (BEM) for thickness ratio greater than 4. But for $T < 4$ the curvature part of the BEM tends to go to zero. However, the uniform strain part (ε_0) brings the total strain to the free strain value, similarly to the case for the other models. The strain energy model overestimated the induced strain in the structure. This is attributed to the forced boundary conditions assumed in the model.

At the same time, the USM gives underestimates compared with the induced strain model, mainly because of the limitation in the model due to the ignorance of the flexural effect of the beam.

Figure 3 shows the induced strain distribution in the beam as predicted by these five models. The USM predicted an average positive uniform strain in the beam, as if the actuator was embedded at the middle of the beam. All other models, except the strain energy model, predict positive strain at the top surface and negative values at the bottom surface. The strain energy model gives an overstrained distribution because of the forced lower boundary condition, zero strain. The pin force model and BEM gave similar distributions with different slopes. In addition, the pin force model agrees with the enhanced pin force model in predicting the same strain at the lower surface of the beam.

The accuracy of these models can be verified experimentally. Methods such as experimental photoelasticity investigations can be used for that purpose.

8. Conclusions

This note reviews the modelling of one-dimensional adaptive structures with surface-bonded piezoelectric actuators using the mechanics of materials approach. Because of the similarity in their approaches and assumptions, the predictions of the pin force model and the BEM are found to be very close to each other and to the finite-element prediction.

References

Alghamdi A A A and Dasgupta A 2000 Eigenstrain techniques for modeling adaptive structures: active damping *J. Intell. Mater. Syst. Struct.* **11** 631–41

Bailey T and Hubbard J E Jr 1985 Distributed piezoelectric-polymer active vibration control of a cantilever beam *AIAA J. Guidance, Control Dyn.* **8** 605–11

Chaudhry Z and Rogers C A 1994 The pin-force model revisited *J. Intell. Mater. Syst. Struct.* **5** 347–54

Crawley E F and de Luis J 1987 Use of piezoelectric actuators as elements of intelligent structures *AIAA J.* **25** 1373–85

Wang B-T and Rogers C A 1991 Modeling of finite-length spatially-distributed induced strain actuators for laminated beams and plates *J. Intell. Mater. Syst. Struct.* **2** 38–58